Dr. Alpo Ranta-Maunus
Research professor
VTT Technical Research
Centre of Finland
Espoo, Finland

Relation of theoretical structural reliability to safety and safety factors of timber structures

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Summary

Structural reliability, expressed in terms of probability of failure or reliability index, is a useful tool for code calibration purposes. It is, however, important to bear in mind that the numerical results obtained are sensitive to input information like distribution functions. Without standardisation of this input data together with target reliability values, there is a great risk of misuse of the reliability analysis. Here a proposal is given for a standard format of code calibration.

Failure statistics are analysed and the damage caused by failures is weighed against the cost of using a higher safety factor. It is concluded that raising safety factors is a highly inefficient way to improve safety: during the observation period 1980 – 2000, a 10% higher safety margin in timber structures would have incurred at least 200-fold higher costs compared with the benefit of having fewer failures in Finland. Accordingly, it is suggested that the present safety level in the building code is fully adequate, and other measures should be used to avoid the gross errors that may lead to failures of structures.

1 Introduction

The use of structural reliability analysis in code calibration is an old topic, but it is alive in Europe because our building code system is gradually changing to a single European code (Eurocode) instead of several national codes. However, the safety level (specified safety-related factors) can be determined in each country separately. When doing so, national authorities are tempted to "improve safety" by increasing the safety margin. This is especially true, when structures have collapsed, and the cases are reported in the main media.

Reliability analysis has been used for the calibration of safety factors in structural codes since the 1970s. The early calculations used normal distribution for loads and lognormal for strength [1]. The acceptable safety level was determined by comparing the results to the experience gained in the past. Since that time we have learned more about the strength and load distributions, and often natural loads are said to follow rather the extreme distributions than normal distribution. The strength of sawn timber seems to better follow the Weibull or normal distribution than lognormal. The selection of distribution function obviously has a remarkable effect on the result of reliability analysis. The change of the dominating variable load distribution from normal to Gumbel distribution, other factors being unchanged, will increase the height of timber beams by 10 to 20%, or if the dimensions are not increased, the change of safety index β from 4.7 to 4.0 [1].

Modern societies wanting to provide safety for all citizens may come under pressure to raise nominal safety levels, even if the relation of the target safety index to the safety of citizens is unclear. The aim of this paper is to analyse the effect of selection of the target safety level and distribution functions on the needed safety factors, and to draw conclusions supported by failure statistics. The failure cases taken place in Finland during past 20 years have been analysed by classifying the incidents into two categories: those which could have been avoided by using a higher safety margin, and to those which are not dependent on the safety factor. By estimating the cost of the damage of the cases belonging to the first category, and comparing it to the cost of using more material in all timber construction, a comparison can be made. This method can be used when the cost of the present safety level is compared with the cost of a higher safety level. Obviously, no conclusion can be made concerning the behaviour of the buildings, if the safety factor would be lower.

This paper is part of a discussion on appropriate target safety levels in timber construction. There are lot of published papers available on this topic. Unfortunately, it was not possible to make reference to these numerous papers, which really would deserve it. This paper is mainly based on results published in [2].

Work in the area of learning from failures has intensified because of the recent failures. In the Nordic countries there are national efforts, which, hopefully, will work in close collaboration for our common benefit in the future.

2 Structural reliability calculation

2.1 Reference case definition

Our reliability analysis uses the design equation as defined in the Eurocode for timber structures:

$$\gamma_G \sigma_{Gk} + \gamma_Q \sigma_{Qk} \le \frac{k_{\mathrm{mod}} f_{0.05}}{\gamma_M} \tag{1}$$

where $f_{0.05}$ is the 5% fractile of the strength distribution, k_{mod} is the modification factor for load duration which is here taken as a known constant, σ_k is the stress caused by the characteristic load, the fractile of the characteristic load being different for permanent (σ_{Gk}) and variable (σ_{Qk}) loads, γ_G and γ_Q are the partial safety factors for the loads, and γ_M is the partial safety factor for material strength. The ratio of variable load to the total load is denoted by

$$\alpha = \sigma_{Qk} / \left(\sigma_{Gk} + \sigma_{Qk}\right) \text{ and } \sigma_k = \sigma_{Gk} + \sigma_{Qk} \tag{2}$$

The performance function is then

$$g = \frac{k_{\mathrm{mod}} f_{0.05}}{\gamma_M} - (1-\alpha)\gamma_G \sigma_k - \alpha \gamma_Q \sigma_k \ge 0 \tag{3}$$

The probability of failure P_f is calculated using the numerical procedure described in [2]. Iteratively, γ_M needed for the adopted target value of P_f is calculated. The reliability index β is calculated from P_f using the usual assumption of normal distribution:

$$P_f = P(g \le 0) = \Phi(-\beta) \tag{4}$$

Table 1 summarises the variables used in the calculation of the reference case, which is close to the present Finnish code. In this computation all statistical variability is included in strength and load functions, and no additional provision is made for inaccuracy in the calculation model or dimensions. As to the values in Table 1, load functions correspond to the present understanding of permanent and natural loads, and the strength distribution is a compromise between sawn timber and engineered wood products.

The calculation results are shown in Figure 1. Quite a linear relationship is observed between γ_M and β, the slope being dependent of α. It can be concluded that $P_f = 10^{-5}$ (β= 4.3) is obtained when γ_M = 1.2 for lightweight structures with α = 0.8. The sensitivity of these results to the change of a single variable is discussed in the next paragraphs.

	Distribution	COV [%]	Fractile [%]	Value
σ_{Gk}	Normal	5	50	
σ_{Qk}	Gumbel	40	98	
$f_{0.05}$	Lognormal	20	5	
γ_G				1.2
γ_Q				1.6
α				0.2, 0.5, 0.8

Table 1: Definition of the reference case

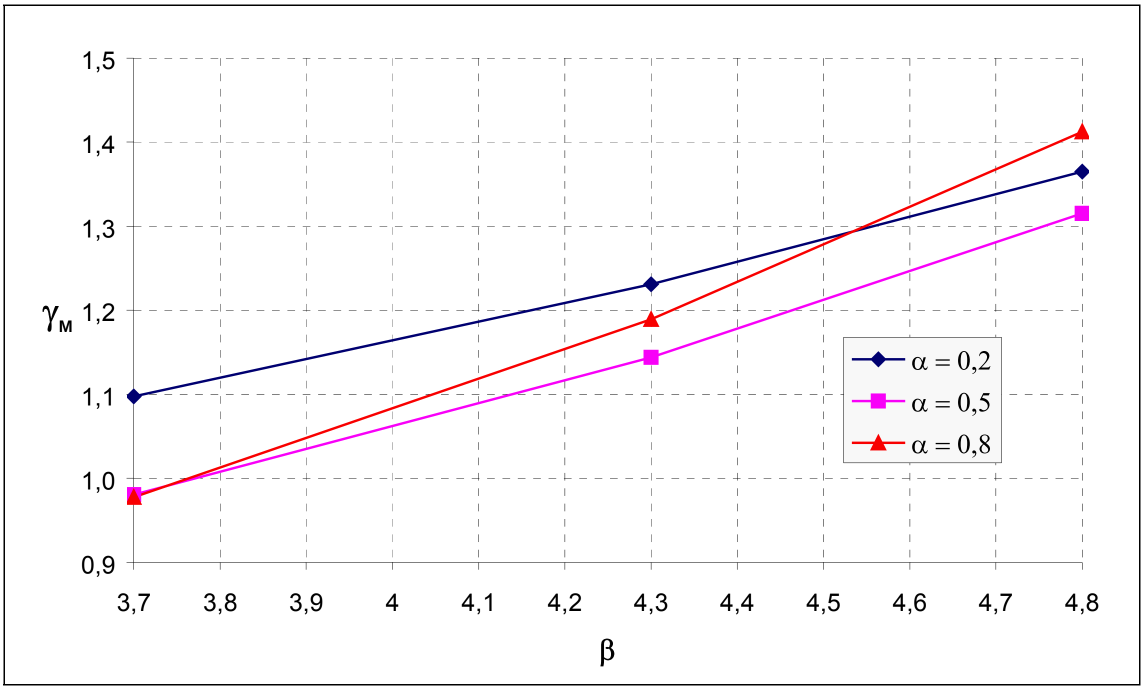

Figure 1: Relation of γM and β for reference time of 1 year in the case defined in Table 1. Timber structures lie between lines α = 0.5 and 0.8

2.2 Sensitivity to distribution functions

The sensitivity of the reliability calculation is analysed by changing a variable from the reference case, one at a time, and comparing γ_M needed for the target reliability. The results are shown in Table 2, and are summarised as follows:

- Case a studies the effect of the COV of the permanent load being 10% instead of 5%: this has no practical effect on timber structures with α > 0.5.
- Case b, the change of variable load distribution from Gumbel to normal distribution and maintaining COV = 0.4, would result in 10 to 20% lower γ_M in cases relevant to timber structures, whereas no change occur with heavy structures. The change is greatest for the highest target safety index.
- The effect of different COV values of strength has been studied in cases c and d. Sensitivity to COV is higher for heavy structures (small α) and at a high safety level. If the target β is 3.8 and α = 0.8, exactly the same value of γ_M is obtained for COV = 10% or 20%, and only slightly higher for 30%. When the target β is 4.8 and α = 0.8, we obtain γ_M = 1.34 for COV = 10% and γ_M = 1.61 for COV = 30%.

- Case e with 2-parameter Weibull distribution results in much higher values for γ_M which also depend strongly on β and α.

- Cases f and c+f correspond to the load safety factors proposed in the Eurocodes. These results are illustrated in Figure 2 (right), and show that different γ_M values are obtained depending on α. This means that the same safety level cannot be achieved with heavy and light weight structures. Rather this could be obtained by the combination of factors shown in the reference case and illustrated on the left side of Figure 2.

Case	Target β	Difference from reference case (value in ref. case)	γ_M for α = 0.2	γ_M for α = 0.5	γ_M for α = 0.8
ref.	3.7 4.3 4.8		1.10 1.23 1.36	0.98 1.14 1.32	0.98 1.19 1.41
a	3.7 4.3 4.8	COV of permanent load 10% (5%)	1.14 1.29 1.43	0.99 1.15 1.33	0.98 1.19 1.41
b	3.7 4.3 4.8	Variable load Normal (Gumbel)	1.11 1.24 1.37	0.96 1.08 1.21	0.88 1.01 1.14
c	3.7 4.3 4.8	COV of strength 10% (20%)	0.93 1.00 1.07	0.93 1.06 1.19	0.98 1.16 1.34
d	3.7 4.3 4.8	COV of strength 30% (20%)	1.32 1.56 1.80	1.11 1.34 1.59	1.05 1.31 1.61
e	3.7 4.3 4.8	Strength Weibull (lognorm) COV=20%	2.08 3.09 4.60	1.64 2.44 3.63	1.37 2.03 3.02
f	3.7 4.3 4.8	γ_G = 1.35 (1.2) and γ_Q = 1.5 (1.6)	1.02 1.14 1.27	0.96 1.12 1.29	1.01 1.23 1.46
c+f	3.7 4.3 4.8	γ_G = 1.35 (1.2) and γ_Q = 1.5 (1.6) and COV of strength 10% (20%)	0.86 0.93 0.99	0.91 1.04 1.17	1.01 1.20 1.39

Table 2: Calculated γ_M –values for three target safety levels $P_f = 10^{-4}$ ($\beta \cong 3.7$), $P_f = 10^{-5}$ ($\beta \cong 4.3$), and $P_f = 10^{-6}$ ($\beta \cong 4.8$).

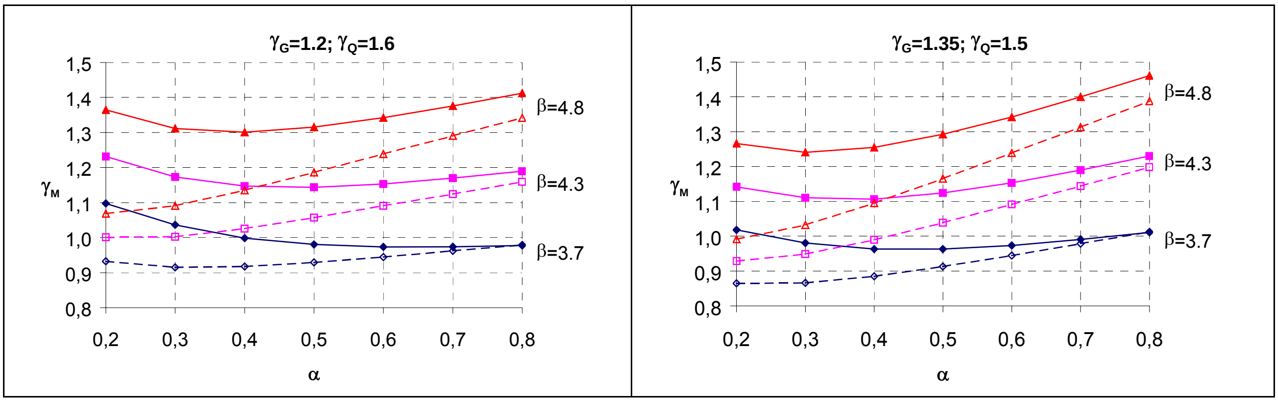

Figure 2: Calculated γ_M values for three target safety index levels as a function of load ratio α. Solid curves are for COV of strength = 20% (lognormal) and broken lines for COV = 10%. Two sets of partial factors for load are used: the reference case of this study (left) and the Eurocode (right).

Load distributions were selected to be Normal when computers were not on the present level. In the Nordic countries the target value of β = 4.8 was adopted at that time, based on a Normal distribution of loads and lognormal distribution of resistance [5].

Later, it has been observed that natural loads (snow, wind) can be more accurately modelled by extreme distributions. Gumbel distribution has been adopted quite commonly. This has 2 consequences:

- Lower beta values are obtained for the same structures (4.0 vs. 4.8, see Fig.2).
- The result of analysis is less sensitive to the choice of parameters.

The latter fact is positive, because the input parameters are either not exactly known or their values are known to vary in the region where the result is applied. Based on this, the use of Gumbel distribution for live loads should be standardised, and, at the same time, the target safety index adjusted on a reasonable level.
Distribution of permanent load is commonly assumed to be Normal. The value and distribution of dead load only has a minor effect on the analysis of light-weight structures, as the timber structures are. This is a relevant question for heavy structures, and is not discussed here.
Selection of a reasonable target β value, the use of Gumbel distribution for live loads, and optimised selection of the ratio of partial factors for permanent and variable loads results in the pleasant situation that the same partial safety factor can be used for materials having COV of a strength not more than 0.20 based on lognormal distribution fitted to lower tail strength values. This would be true in a wide range of structures from heavy to light. This would suggest that practically same material safety factor can be used for steel, concrete and industrial wood products. Sawn timber graded to C30 and lower grades would require a higher material safety factor, say 1.4 vs. 1.2.

2.3 Sensitivity to target reliability level

Target reliability level is often expressed in terms of reliability index β. The Eurocode defines target β = 4.8 for a one-year return period, which is equivalent to 3.8 for a 50-year return period. This is for regular structures; other values are given if the consequences of collapse of the structure are especially high or low.
The Swedish building code defines three safety classes of buildings, and adopts target β values of 3.7, 4.3 and 4.8 for low, normal and high safety classes, respectively. Consequently, the safety coefficient is multiplied by factors of 1, 1.1, and 1.2, respectively. In the new Danish building code a value as high as β = 4.79 is used as the target value for normal structures.
An international expert group, the Joint Committee on Structural Safety (JCSS), has worked on reliability-based design for years and suggests target values as listed in Table 3. The target β for normal structures is 4.2, and recommendations vary depending on what the cost of higher safety amounts to and what the consequences of failure are.

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Major consequences of failure
Large	3.1 (P_f ~ 10^{-3})	3.3 (P_f ~ 5 10^{-4})	3.7 (P_f ~ 10^{-4})
Normal	3.7 (P_f ~ 10^{-4})	**4.2 (P_f ~ 10^{-5})**	4.4 (P_f ~ 5 10^{-6})
Small	4.2 (P_f ~ 10^{-5})	4.4 (P_f ~ 5 10^{-6})	4.7 (P_f ~ 10^{-6})

Table 3: Tentative target reliability indices β and associated target failure rates related to a reference period of 1 year and ultimate limit states [4]

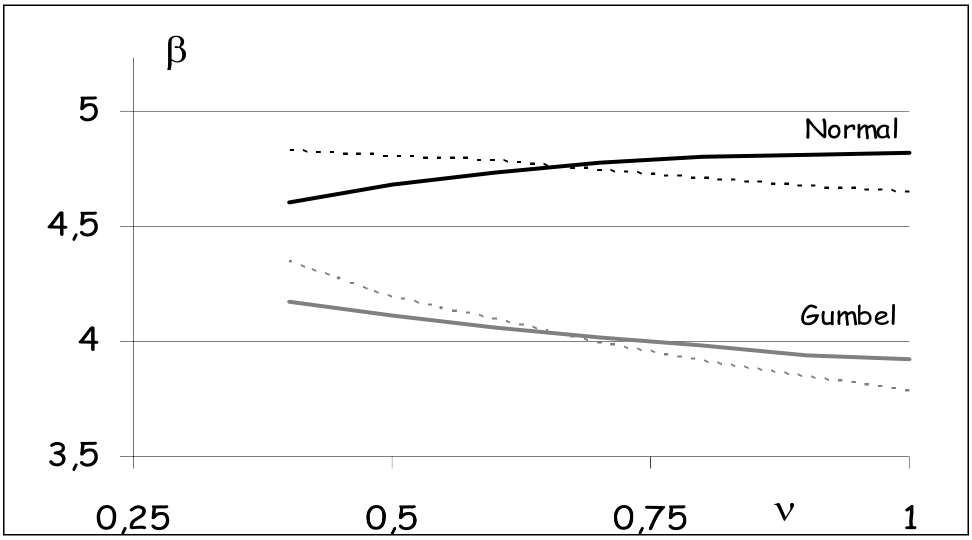

Figure 3: Reliability index levels calculated for the present Swedish code (continuous curves) and old code (dotted curves) when variable loads are Normal or Gumbel distributed.v is the ratio of variable load to total load. From [5]

2.4 Sensitivity to strength distributions

Quite commonly, lognormal distribution is used for the strength of building materials. This is found to be a good selection for man-made materials. For natural materials like sawn timber, Normal or Weibull distributions give a better fit, especially if the material is not strength graded. The grading procedure affects the distribution, depending on the quality of the grading, and the distributions may be different for different grades.
In spite of the exact form of distribution, lognormal distribution can also be used for sawn timber. The correct method of analysis requires that the parameters of lognormal distribution are based on fitting to the lower tail of test values, say 10 or 15 % of the weakest values. This should be done to obtain the correct result, because the lowest strength values have the greatest influence on the reliability value. A consequence of this is that the parameter of lognormal distribution indicating the coefficient of variation will normally be higher than the COV calculated from all the test results. However, the result of reliability calculation in terms of β by this method is as favourable for wood as one can obtain.

3 Failure statistics

Failures of timber structures in Finland during 1980 –1996 have been analysed and summarised [6]. Table 4 includes information based on this report and information for 1997 – 2000 based on VTT`s own files. This information includes the vast majority of all failures to have taken place in Finland over the past 20 years. Eighteen failures are reported, and 16 other cases are included in which the error was observed before collapse, but where economic damage resulted. Most identified failures of timber structures concern roof structures, except for the last case C in Table 4, in which a lack of racking resistance of vertical structures was the reason for collapse, and case B, in which a hanging ceiling was not properly fastened. In most cases the failure is caused at least partly by designer error, but mistakes in erection of the building are often a contributing factor. Not all cases are analysed well enough to reveal the real reasons behind the failure. None of these failures caused injuries but some did pose a significant risk.

In most cases the failure mechanism was some kind of instability, often buckling of the upper chord of a truss caused by lack of bracing. Frequently these are in areas where the shape of roof is complex, and may additionally have an increased local snow load not taken into account by the designer.

Most structural failures covered by one or more of following reasons:

- **Loss of stability**. Importance of stability of structural members is not understood by builders on site: compression members are not supported as required or racking resistance of the building is neglected. Design errors are not common.
- **Moisture in wood**. Wood is wet for different reasons: leakage, vapour barrier lacking, rain during construction. Various problems result: rot, low compression strength, cracking.
- **Inexperienced wood designer**. Failure mode of splitting of wood is forgotten. Weakness of wood in perpendicular to grain direction is not obvious to designers having basic education mainly in steel construction. Moisture loads can contribute to this failure mode by shrinkage.

Case No.	Structure responsible	Failure mechanism / location	Effect of safety factor	Economic damage [1000 €]	Notes
1	Nailed roof truss	Instability	No	375	Total: 15 similar repairs
2	Nail plate truss	Instability	No	100	
3	Nailed roof truss	Not identified	Some	10	Load exceeded code value: ice
4	Glued trusses	Instability	No	20	
5 nc	Nail plate truss		Some	10	Many errors observed during construction
6 nc	Nail plate frame	Large deflection	No	10	
7 nc	Nail plate truss	Instability	No	50	
8	Nail plate truss	Instability	No	500	
9	Nailed roof truss	Connection	No	100	
10	Nail plate truss	Instability	Some	200	Combination of many errors
11	Nail plate truss	Connection	No	200	Combination of many errors
12	Nail plate truss	Instability	No	200	Combination of many errors
13 nc	Glulam arch	Degradation	No	400	3 similar cases, exposed
14 nc	Nail plate trusses	Instability	No	80	9 similar cases
15a	Glulam beam	Beam failure	Some	10	Load exceeded code value
15b	Glulam beam	Beam failure	No	10	Degradation, leakage
15c	Glulam beam	Beam failure	Some	10	Load exceeded code value
15d	Glulam beam	Support area	Some	10	Design error
15e	Glulam beam	Beam failure	No	10	Combination of many errors
18 nc	Nail plate truss	Instability	No	10	Large deflection, no collapse
A	Glulam beam	Connection	No	25	Design error
B	Hanging of ceiling	Connection	No	1200	Combination of many errors
C	Walls	Instability	No	500	2 cases, lack of racking resistance
Total			all	5040	
			some	250	

Table 4: Summary of failures of timber structures in Finland 1980 – 2000 including damage cases in which repairs were made and no collapse occurred ("nc" after case No.).

These 34 damage incidents were classified, by engineering judgement based on the documentation available, as cases in which a higher safety factor might have prevented failure, or as those in which a moderate increase of the safety margin would have had no effect. In Table 4 “some” in the “effect of safety factor” column indicates cases in which a 10% higher safety factor might have prevented failure, even though design errors were identified in all cases but one.

The total economic damage of all cases was 5 million €, including the cost of repairs. In those cases in which a moderately higher safety factor might have prevented the damage, the total loss was 0.25 million €. Of the 18 roofs collapsed, five might not have collapsed had the safety factor been higher. However, there is no certainty that any of the failure cases would have been avoided by the use of higher safety factor.

In Finland in 1980 – 2000, about 1.4 million cubic metres of strength-graded sawn timber was used in prefabricated roof trusses alone, and 0.5 million cubic meters of glulam was used in construction. If the safety factor had been 10% higher, the cost in trusses would have been at least 25 million € higher, and in glulam 15 million €. If we count only 10 million € for other timber structures, we can roughly estimate that the adoption of a 10% higher safety factor in 1980 would have required 50 M€ more investment in timber construction, whereas the savings from less damage would have been only 0.25 M€ or less. This comparison is a rough estimate, not only because the failure analysis has not been made carefully in all cases, but also because structures built in 1980 – 2000 may fail later when high snow loads develop. On the other hand, these cases include also failures in buildings built before 1980. However, an obvious conclusion at the moment is that it would have been a bad investment to use 50 M€ for a gain of maximum 0.25 M€.

4 Conclusions

4.1 Target safety level

The failure statistics of timber structures in Finland suggest that the safety level of the present building code is adequate: a 10% higher safety margin would have incurred at least 200-fold higher costs compared with the benefit of having fewer failures. Most failures would have occurred independently of the change in safety margin. Accordingly, other measures than an increase of the safety margin in design are more effective means to increase public safety. Measures have been taken to co-ordinate the work of the main designer of the building and the engineers who calculate the strength of prefabricated elements.

The present safety level can be characterised as follows: the partial factors for loads are γ_G = 1.2 and γ_Q = 1.6, and for material γ_M = 1.3. If case d in table 2 is used as the most appropriate for timber structures, we obtain β = 4.2 (annual) for typical timber structures, when no provisions are made for model accuracy and dimension tolerances. Based on failure statistics, this seems an adequate target level for structural safety. A higher target safety level has also other disadvantages: it makes the output of code calibration more sensitive to variables as COV of material strength and the load ratio α. Because the code is intended to cover different materials and buildings, the nominally high safety level is becoming uneven and makes it difficult to determine γ-factors in a balanced way. Also the benefit of nominally high safety level is questionable, because the parameters used in the calculation are not known to the adequate accuracy.

4.2 Proposal for code calibration

When structural reliability analysis is used for code calibration or direct dimensioning of structures, certain variables need to be standardised in order to obtain useful results. Knowing that there are international groups working with the standardisation of structural reliability analysis (JCSS, COST E24), here is a suggestion for timber structures to facilitate the discussion:

- Variable loads: Gumbel distribution. If no regional information is available, COV = 0.4 should be used for snow and wind loads. COV = 0.2 can be used for floor loads.
- Permanent loads: Normal distribution. COV = 0.1 (0.05 or 0.1 gives practically the same result).
- Strength: Lognormal distribution to be fitted to the lowest 15% of values (min. 75 values, total population 500).
- Other factors, like model or dimension accuracy, can be taken as Normal distributions. In the case of sawn timber structures, these have a minor effect.
- Target reliability level $\beta = 4.0$ is adequate with the selections above and would result in dimensions of structures similar to those we have in the Nordic countries today.

There are other factors, depending on load duration, moisture content and size of structural members, to be considered in the design of timber structures. Unless there is new and comprehensive information on these issues, they should be taken as deterministic factors according to the code of concern.

References

[1] Svensson, S. and Thelandersson, S. "*Aspects on reliability calibration of safety factors for timber structures*", CIB-W18 Meeting thirty-three, Paper 33-1-1, 2000.

[2] Ranta-Maunus, A., Fonselius, M., Kurkela, J. and Toratti, T. *Reliability analysis of timber structures,* VTT Research Notes 2109, Espoo, 2001.

[3] Ranta-Maunus A., "*Target Safety Level in Reliability Analysis of Timber Structures*", Proceedings of WCTE 2002 Conference, Shah Alam, 2002.

[4] Joint Committee on Structural Safety, Probabilistic model code, Part 1 – Basis of design. 12th draft, internet version: http://www.jcss.ethz.ch, 2000.

[5] Svensson S., Thelandessson S., "*Aspects on reliability calibration of safety factors for timber structures"*, Holz als Roh- und Werkstoff, Vol.61, No.5, 2003, pp.336-341.

[6] Törmänen, J. and Leskelä, M.V. *Summary of failures in Finnish timber structures 1980 –1995* (in Finnish), Tutkimusraportti RTL 0021, Oulun yliopisto. 60 p. ISBN 951-42-4509-1, 1996.