Investigation of lateral torsional buckling of timber beams subjected to combined bending and axial compression

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1. Motivation

Due to the increasing demand for long-spanning material-efficient timber structures, the design of slender members is getting more and more important. Trussed beams for example are ideal for big spans because of their low and efficient material consumption (Figure 1). These structures are typically loaded by a combination of normal force, shear force and bending moment. While short beams reach the maximum load bearing capacity due to exceeding of the maximum tensile and compressive strength (cross-section failure), stability failure is usually dominant for slender members. This is characterized by excessive deformations. And the resulting additional internal forces due to imperfections and effects of second-order theory, that may exceed the load bearing capacity. Latest investigations [1] to [4] have shown that the design methods provided by Eurocode 5 [5] for lateral torsional buckling of timber beams subjected to combined bending and compression tend to be conservative and do not allow an economic design.

Eurocode 5 provides two design concepts for slender timber members, where one is the effective length method and the other is one based on calculations of second order theory. In both methods the material specific behaviour of timber is highly simplified. By taking the elasto-plastic material behaviour under compression and the size effect on the tensile strength into account, high reserves of the load-bearing capacity can be activated for slender timber beams.

This material specific behaviour of timber can be considered either in numerical (FEM) or analytical models. The generated results can be used for the verification of the design formulas of EC5 [5]. Analytical models have proven to be very powerful in the past [6],[7] and are also the basis of the effective length method for lateral torsional buckling in current EC5 [8].

However, the mentioned and significant material specific behaviour of timber has not yet been adequately captured with these models.

Figure 1: Trussed beams subjected to bending and compression, Lignotrend Produktions GmbH [9]

In this paper the results of a master thesis at the University of Stuttgart [4], in which an analytical calculation method for the investigation of these effects on slender and especially medium slender timber members was developed, are presented.

Within this newly developed calculation method, stability verification for lateral torsional buckling according to second order theory is combined with approaches, which take into account the plasticizing of timber under compression and the size effect on the tensile resistance.

In this paper it is shown that plasticizing under compression and the size effect have a significant impact on the load bearing resistance of imperfection-sensitive timber structures. With the developed and in this paper presented analytical model it can be shown that the effective length method is partly very conservative, while verifications based on calculations according to second order theory provide good approximations to the load bearing capacity with even potential for optimization.

2. Calculation proceeding

2.1. Material specific behaviour

For modelling the material behaviour of timber, the mechanical models described by *Hörsting* [3] are used. There both, plasticizing of timber under compression and size effect on the tensile strength, are included when calculating the load-bearing capacity of timber beams subjected to bending and axial compression.

Plasticizing under compression

Figure 2 shows the assumed idealized stress-strain behaviour of timber due to axial loading for the analytical calculations. For axial compression an elasto-plastic behaviour is assumed, meaning the cross-section starts plasticizing when the compressive strength is reached. There is no failure criterion implemented for compression. For axial tension, which occurs in this study due to bending and effects of second order theory, the behaviour is assumed to be linear elastic with brittle failure when reaching the tensile strength.

According to *Hörsting* [3] the internal forces N_x, M_y, M_z of a cross-section can be calculated when assuming a perfect elasto-plastic material behaviour (Figure 2) with the help of equations (1) to (3) on the basis of the relations shown in Figure 3.

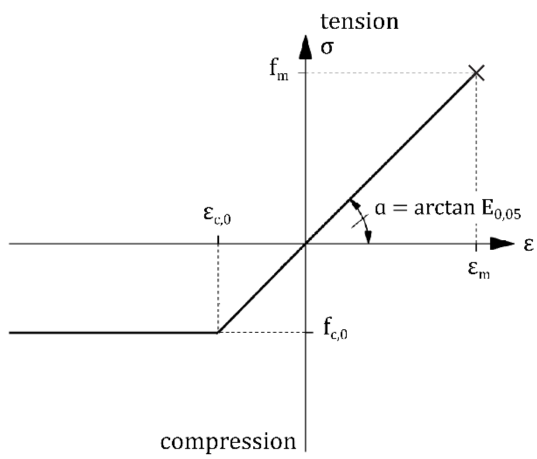


Figure 2: Schematic stress-strain graph of the assumed simplified material behaviour of timber due to axial loading [3]

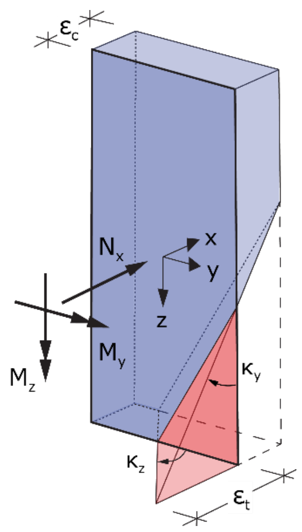


Figure 3: Curvatures (κ_y, κ_z) and elastic strains (ε_c, ε_t) for a random strain distribution due to combined bending and axial compression (M_y, M_z, N_x)

$$N_x = EA \cdot \left(\varepsilon_t - \kappa_y \frac{h}{2} - \kappa_z \frac{b}{2}\right) + \frac{E}{6\kappa_y\kappa_z} \cdot \begin{pmatrix} +\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y - b\kappa_z)\rangle^3 \\ -\langle -(\varepsilon_t - \varepsilon_c - b\kappa_z)\rangle^3 \\ -\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y)\rangle^3 \end{pmatrix} \quad (1)$$

$$M_y = EI_y\kappa_y + \frac{E}{24\kappa_y^2\kappa_z} \cdot \begin{pmatrix} -\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y - b\kappa_z)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - h\kappa_y - b\kappa_z) \\ +\langle -(\varepsilon_t - \varepsilon_c - b\kappa_z)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - 2h\kappa_y - b\kappa_z) \\ +\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - h\kappa_y) \end{pmatrix} \quad (2)$$

$$M_z = EI_z\kappa_z + \frac{E}{24\kappa_z^2\kappa_y} \cdot \begin{pmatrix} -\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y - b\kappa_z)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - h\kappa_y - b\kappa_z) \\ +\langle -(\varepsilon_t - \varepsilon_c - b\kappa_z)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - b\kappa_z) \\ +\langle -(\varepsilon_t - \varepsilon_c - h\kappa_y)\rangle^3 \cdot (\varepsilon_t - \varepsilon_c - h\kappa_y - 2h\kappa_z) \end{pmatrix} \quad (3)$$

with
b width of cross-section (see Figure 5)
h height of cross-section (see Figure 5)
A cross-section area (h x b)
I_y moment of inertia in direction of y
I_z moment of inertia in direction of z
E modulus of elasticity in longitudinal direction
ε_t elastic tensile strain (see Figure 3)
ε_c elastic compressive strain (see Figure 3)
κ_y curvature around the strong y-axis (see Figure 3)
κ_z curvature around the weak z-axis (see Figure 3)
$\langle\ \rangle$ Föppl-symbol

Size effect on the tensile strength

Timber is subject to a size effect due to its distinctive inhomogeneity like irregularities in growth and defects, which lead to deviations to the ideally straight grain orientation [10]. The more defects there are, the more sensitive timber is to brittle tensile failure; and the larger the stressed timber volume is, the higher the probability of local defects within the stressed volume is. Since the tensile stressed volume of a cross-section in the case of pure bending is smaller than in the case of pure tension, the tensile strength of timber in pure tension is lower due to the size effect. Additionally, the stress distribution within a cross-section plays an important role for the size effect, due to the probability of maximum stress and defects coming together. The size effect is taken into account by modifying the tensile strength throughout an iterative calculation. Here, the Weibull distribution [11] is used to describe the relation of tensile and bending strength (eq. 4). The distribution parameter k_h of the Weibull distribution can be determined by solving equation (4) for k_h.

$$f_{t,0} = \left(\frac{1}{2+2k_h}\right)^{\frac{1}{k_h}} \cdot f_m \tag{4}$$

with $f_{t,0}$ tensile strength parallel to the grain
f_m bending strength
k_h distribution parameter

With the parameter k_h a modified maximum elastic tensile strain $\varepsilon_{t,mod}$ on the basis of the size effect can be determined iteratively using the cross-section curvatures and solving the following equation for $e_{t,mod}$.

$$\varepsilon_m = \left[2 \cdot \frac{\varepsilon_{t,mod}^{(2+k_h)} - \langle \varepsilon_{t,mod} - \kappa_y h \rangle^{(2+k_h)} - \langle \varepsilon_{t,mod} - \kappa_z b \rangle^{(2+k_h)} + \langle \varepsilon_{t,mod} - \kappa_y h - \kappa_z b \rangle^{(2+k_h)}}{A \cdot (2+k_h) \cdot \kappa_y \cdot \kappa_z}\right]^{\frac{1}{k_h}} \tag{5}$$

with ε_m elastic bending strain
$\varepsilon_{t,mod}$ modified elastic tensile strain

Therefore, the modified tensile strength $f_{t,mod}$ can be calculated with

$$f_{t,mod} = \varepsilon_{t,mod} \cdot E \tag{6}$$

2.2. Calculation of internal forces

The following investigations are performed on imperfection-sensitive single span beams with fork bearings under combined bending moment around the strong axis and normal force (see Figure 4).

If it is assumed that the beam shown in Figure 4 is subjected to geometrical imperfections (Figure 5), the stability problem can be translated into a pure stress based mathematical

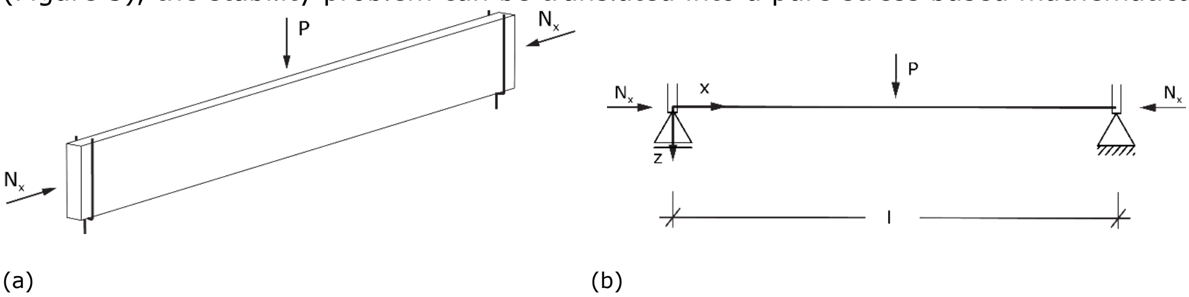


Figure 4: Three-dimensional (a) and two-dimensional (b) structural system of a timber beam with fork bearings, subjected to a compressive force N_x and a bending moment $M_{y,I}$ due to a single force P at midspan

problem. Due to imperfections (initial torsion ϑ and initial curvature in y-direction e_y) an additional internal bending moment around the weak z-axis $M_{z,II}$ occurs. An interaction of the initial curvature in y- and z-direction is neglected hereafter and $e_z = 0$ assumed. Equations (7) and (8) describe the internal forces due to second order effects, noting, that compressive forces are assumed positively.

$$M_{z,II} = \frac{M_{y,I} \cdot \vartheta + \left(N_x + N_{z,crit} \cdot \alpha_M^2\right) \cdot e_y}{1 - \alpha_{N,z} - \alpha_M^2} \tag{7}$$

$$M_{y,II} = \frac{M_{y,I} + N_x \cdot e_z}{1 - \alpha_{N,y}} \tag{8}$$

with $M_{y,I}$ external bending moment due to P (see Figure 4)

$M_{y,crit}$	critical bending moment
N_x	axial force (compression positive, see Figure 4)
$N_{y/z,crit}$	critical axial force in direction of y/z
$\alpha_{N,y/z} = N_x/N_{y/z,crit}$	factor of second order amplification for flexural buckling
$\alpha_M = M_{y,I}/M_{y,crit}$	factor of second order amplification for lateral torsional buckling
e_y	initial curvature in y-direction
$e_z = 0$	initial curvature in z-direction (neglected hereafter)
ϑ	initial torsion

2.3. Failure criterion

The following calculations are based on the assumption that a global failure of timber members at risk of lateral torsional buckling generally occurs when the tensile strength is exceeded in the tensile zone (Figure 6). If the compressive stresses reach the compressive strength, the cross-section begins plasticizing.
Equation (9) describes the failure criterion. It is to be noted that the compressive axial load N_x is taken into account with a negative sign in order to consider the favourable impact on the tensile stress verification.

$$-\frac{N_x}{A} + \frac{M_{y,II}}{W_y} + \frac{M_{z,II}}{W_z} \leq f_{t,mod} \tag{9}$$

2.4. Analytical calculations

A flow chart for the stress verification based on an analytical calculation of second order bending moments is shown in Figure 7. With this procedure a maximum bending moment $M_{y,I}$ for a timber beam of specific dimensions and axial loading can be determined.
First, depending on the input data of cross-section dimensions and material parameters, the critical bending moment $M_{y,crit}$ is calculated according to EC5 [5]. Hereby, the assumed structural system and the type of loading are considered by the effective length $l_{ef,m}$. In the next step the magnitude of the axial compression load N_x is specified. With the help of the failure criterion (eq. (9)), the maximum bending moment $M_{y,I}$ can be calculated. Therefore, by taking into account the geometric imperfections e_y, e_z and ϑ, the bending moments $M_{y,II}$ and $M_{z,II}$ can be determined iteratively. As a result, there is a M-N combination with a maximum bending moment for a given axial compression load, which takes into account an elasto-plastic material behaviour under compression and satisfies the failure criterion of equation (9).

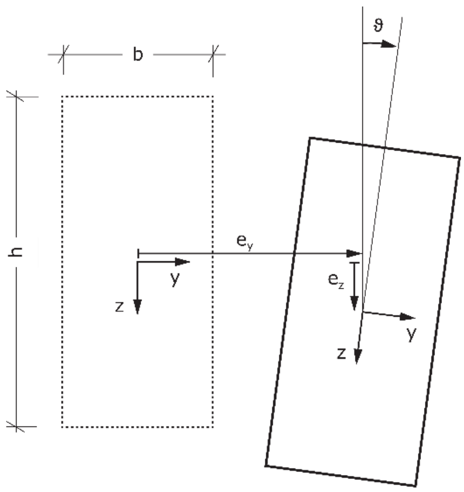


Figure 5: Cross-section of beam shown in Figure 4; imperfections at midspan in form of initial torsion ϑ and initial curvature in y- and z-direction (e_y, e_z)

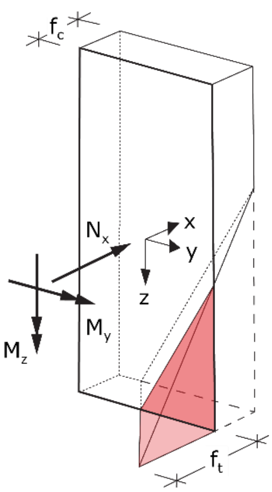


Figure 6: Plot of a random three-dimensional stress distribution due to bending and axial compression

For an additional consideration of the size effect, further steps are necessary. Based on the bending moments $M_{y,II}$ and $M_{z,II}$ the curvatures κ_y and κ_z can be determined. Knowing the curvatures, a modified maximum elastic tensile strain $\varepsilon_{t,mod}$ (eq. (5)) and the modified tensile strength $f_{t,mod}$ (eq. (6)) can be calculated. Now the failure criterion can be reevaluated considering $f_{t,mod}$. The size effect has to be taken into account within an iterative process. The described steps are repeated until an abort criterion is reached and the tensile strength does not increase notably anymore.

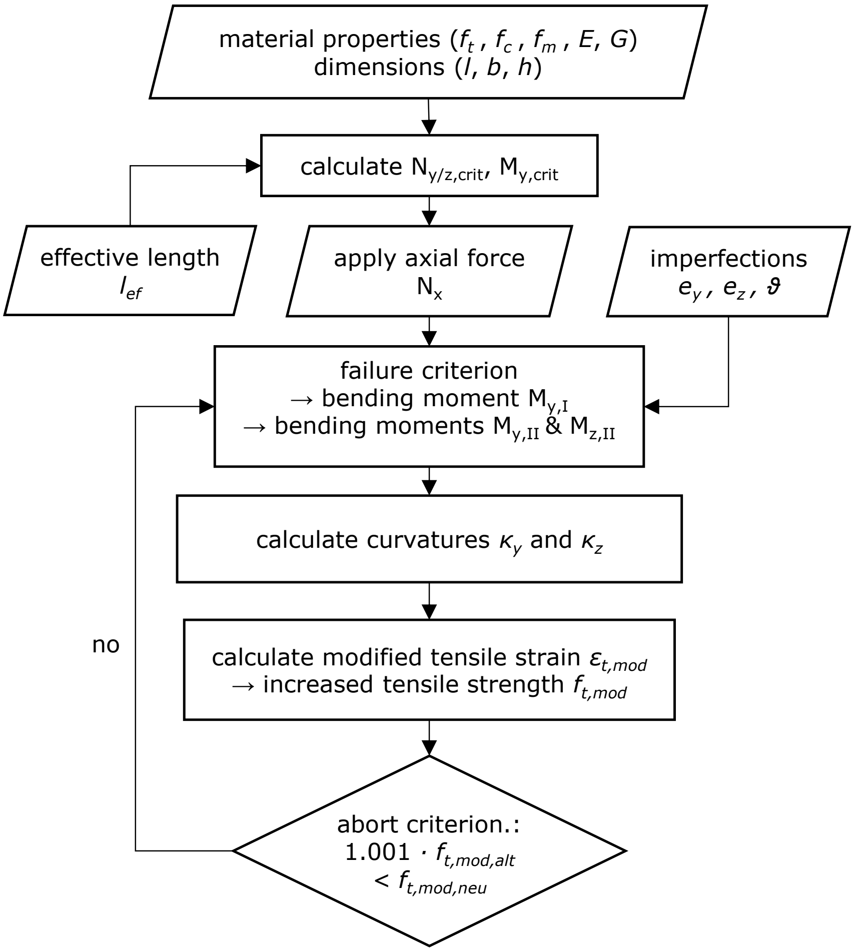


Figure 7: Flowchart for determining the maximum bending moment of a timber beam for a given axial force while considering elasto-plastic behaviour under compression and the size effect

3. Parametric study

3.1. Overview

In general, the parametric study is intended to gain knowledge about the influence of the elasto-plastic material specific behaviour and the size effect on the load-bearing capacity of timber beams at risk of lateral torsional buckling. The following parameters have been investigated:

- Relative slenderness ratio $\lambda_{rel,m}$
- Ratio of axial compression force N_x to bending moment $M_{y,I}$
- Type of timber (glued laminated timber; solid timber)

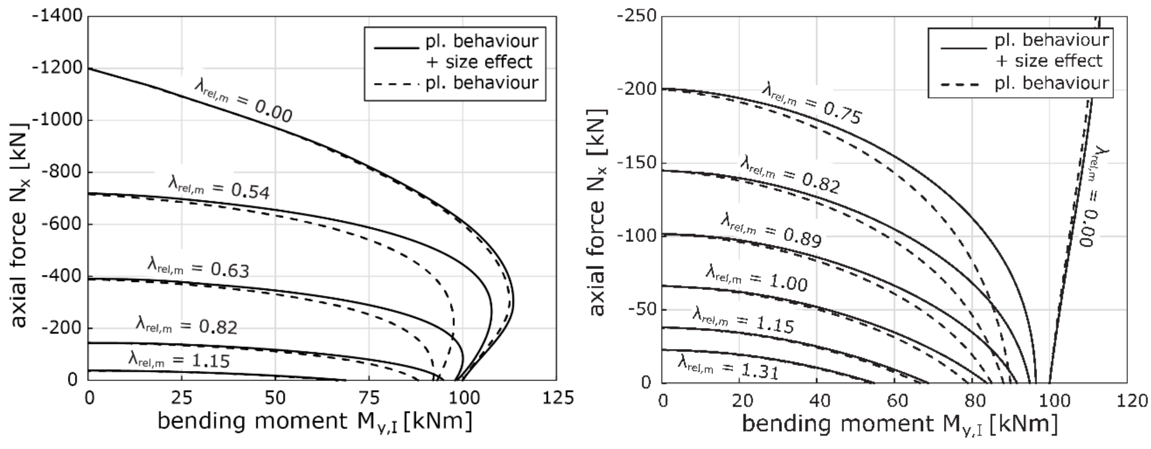
The cross-sectional dimensions of the investigated timber beams are assumed to be 500 mm x 100 mm (h x b). The slenderness is thus varied via the length of beam. Geometrical imperfections are assumed to be e_y = l/400 for initial curvature in direction of the weak axis [5] and $\vartheta_0 = 0.05 \cdot b/h$ for the initial torsion [12]. The structural system and the load application correspond to the system shown in Figure 4, where bending occurs due to a single force applied at midspan. In general, the type of timber is assumed to be glued laminated timber GL 24h according to EN 14080 [13]. Partial factors and modification factors for material parameters are set to 1.0. The assumed material parameters are shown in Table 3.1.

Table 3.1: Assumed material properties for glued laminated timber GL 24h [13] and solid timber C24 [14]

	$E_{0,05}$ [N/mm²]	$G_{0,05}$ [N/mm²]	$f_{m,k}$ [N/mm²]	$f_{t,0,k}$ [N/mm²]	$f_{c,0,k}$ [N/mm²]
GL 24h	9600	540	24.0	19.2	24.0
C24	7400	690	24.0	14.5	21.0

3.2. Influence of slenderness

In order to investigate the influence of slenderness on the load-bearing capacity of timber beams at risk of lateral torsional buckling, the interaction between bending moment and axial compression force for glulam GL 24h is investigated. Figure 8 (a) shows the M-N load carrying capacity for a wide range of slenderness. Figure 8 (b), on the other hand, shows especially beams of intermediate length which range from $\lambda_{rel,m}$ = 0.75 to $\lambda_{rel,m}$ = 1.4. The dashed curve describes the analytical results of the load-bearing capacity with a pure consideration of the elasto-plastic material behaviour, the solid line the load-bearing capacity with an additional implementation of the size effect.
In general, a decrease in load-bearing capacity can be observed with increasing slenderness. For short beams ($\lambda_{rel,m}$ < 0.75) a "bump" forms, which partly exceeds the elastic



(a) (b)

Figure 8: Interaction of bending moment $M_{y,I}$ according to first order theory and axial compression force N_x for timber beams (GL24h) with short (a) and intermediate length (b) depending on the relative slenderness $\lambda_{rel,m}$, based on a verification for stresses of second order theory for lateral torsional buckling

moment carrying capacity of 100 kNm. For beams of intermediate length (Figure 8 (b)), however, the bump flattens out and the impact of plasticizing under compression and the volume effect is decreasing. For $\lambda_{rel,m}$ = 0.75, the maximum increase in the bending moment carrying capacity is approximately 8 % due to an additional consideration of the volume effect. With a slenderness ratio of $\lambda_{rel,m}$ = 1.31, however, the maximum increase is only about 1 %.

3.3. Influence of axial compression force

Figure 9 shows the load-bearing capacity with regard to the bending moment as a function of the slenderness $\lambda_{rel,m}$. Depending on the ratio of compression force N_x/N_0, where N_0 represents the maximum elastic load-bearing capacity ($N_0 = f_{c,0,k} \cdot A$). In addition, the maximum elastic bending moment capacity of the given cross-section ($M_{y,0} = f_{m,k} \cdot W_y$) is shown for comparison (dotted line). The dashed curves again describe the analytical results of the load-bearing capacity with a pure consideration of the elasto-plastic material behaviour, the solid lines the load-bearing capacity with an additional implementation of the size effect.
As the compression force increases, the curves generally drop more steeply achieving a specific maximum slenderness for a specific compressive load. For a pure bending (N_x/N_0 = 0), the graph is always below the dotted line of the elastic bending moment capacity. If, however, a compression force is applied, a "hump" forms due to the material specific behaviour of plasticizing under pressure and the size effect. Especially in the range of short beams and beams of intermediate length, the possible moment carrying capacity $M_{y,0}$ according to the linear elastic stress criterion is exceeded.

3.4. Influence of solid and glued laminated timber

The load-bearing capacity of glued laminated timber GL 24h according to EN 14080 [13] is compared to solid timber C24 according to EN 388 [14]. Depending on the type of timber, there are differences in

- Stiffness properties (modulus of elasticity E, shear modulus G)
- Strength properties (tensile, compressive strength)
- Ratio of strength to stiffness (f_t / E)
- Ratio of compressive to tensile to bending strength (f_c / f_t / f_m)

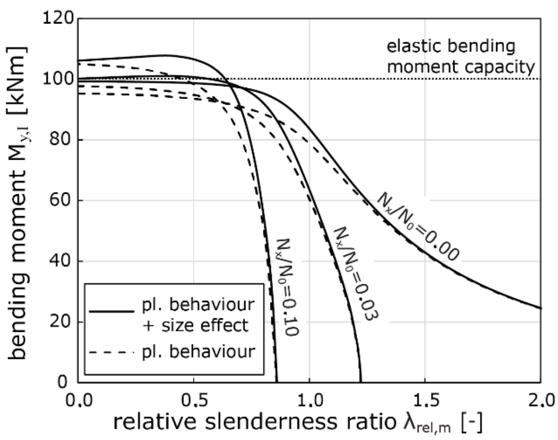


Figure 9: Maximum load bearing capacity with regard to the bending moment capacity as a function of the slenderness depending on different axial compression loadings for glued laminated timber GL 24h with $\lambda_{rel,m}$ relative slenderness of lateral torsional buckling, $M_{y,I}$ maximum bending moment according to first order theory, N_x compression force, $N_0 = f_{c,0,k} \cdot A$ maximum compression force

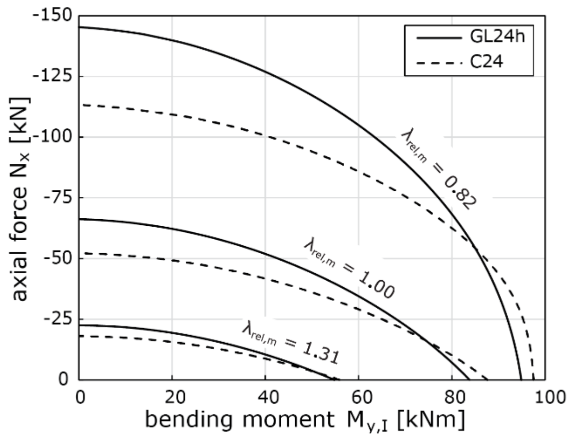


Figure 10: Interaction of bending moment $M_{y,I}$ according to first order theory and axial compression force N_x for glued laminated timber GL 24h in comparison to solid timber C24 depending on the relative slenderness $\lambda_{rel,m}$, based on a verification for stresses of second order theory for lateral torsional buckling

The stiffness properties have a direct effect on the critical bending moment $M_{y,crit}$ and the critical axial force $N_{y/z,crit.}$ Therefore, members with low stiffness tend to fail due to excessive deformation and the resulting additional internal forces more easily.
The ratio of tensile strength to bending strength f_t/f_m essentially determines the influence of size effect [15]. For a smaller f_t/f_m ratio, the size effect is therefore bigger. The compressive strength, on the other hand, has an influence on the axial load bearing capacity in compression because a lower compressive strength results in earlier plasticizing of the cross-section.

Figure 10 shows the load bearing capacity of timber beams with an interaction of bending moment and axial compression force of glued laminated timber GL 24h (solid line) compared to solid timber C24 (dotted line). Three different slenderness ratios ($\lambda_{rel,m}$ = 0.81, 1.00, 1.31) within the range of intermediate slenderness are considered. With the same bending strength (f_m = 24 N/mm²) glulam provides higher stiffness properties and a higher tensile and compressive strength than solid timber.
Across all examined slenderness ratios, gluelam shows a higher axial load bearing capacity compared to solid timber, which results from the higher compressive strength $f_{c,0,k}$. In the case of pure bending, however, solid timber achieves a higher bending moment capacity. Thus, the bending moment capacity of solid timber beams for $\lambda_{rel,m}$ = 0.81 is approx. 3 % higher than for the glued laminated timber beams. With higher slenderness the difference is decreasing. For a slenderness ratio of $\lambda_{rel,m}$ = 1.31 the difference regarding the bending moment capacity is reduced to approx. 1.5 %. The higher bending moment capacity can be explained by the higher influence of the size effect for solid timber with two axis bending $M_{y,II}$ and $M_{z,II}$, because of the smaller f_t/f_m ratio.

4. Comparison with the literature

Presently there are two common design concepts for timber beams at risk of lateral torsional buckling which are both covered in EC5 [5] - the effective length method and design based on calculations of second order theory (see Figure 11). For the described investigations the interaction formula for the effective length method is taken from DIN 1052:2004 [16] due to shortcomings of the formula in current EC5 [1],[18].
The effective length method for lateral torsional buckling of timber beams subjected to both bending and axial compression represents a combination of the two separate effective length methods for pure axial loading (k_c-method) and for pure bending (k_m-method).

	effective length method	second order theory	
elastic behaviour	$\frac{N_x}{k_{c,y}Af_c}+\frac{M_y}{k_m W_y f_m}\leq 1{,}0$ $\frac{N_x}{k_{c,z}Af_c}+\frac{M_y}{k_m W_y f_m}\leq 1{,}0$ (10)	$\frac{N_x}{Af_c}+\frac{M_{y,II}}{W_y f_m}+\frac{M_{z,II}}{W_z f_m}\leq 1{,}0$ (12)	
plastic behaviour		according to EC5: $\left(\frac{N_x}{Af_c}\right)^2+\frac{M_{y,II}}{W_y f_m}+\frac{M_{z,II}}{W_z f_m}\leq 1{,}0$ (13)	according to equation (9): $\frac{-N_x}{Af_m}+\frac{M_{y,II}}{W_y f_m}+\frac{M_{z,II}}{W_z f_m}\leq 1{,}0$ (15)
plastic behaviour + size effect	$\frac{N_x}{k_{c,y}Af_c}+\frac{M_y}{k_m W_y f_m}\leq 1{,}0$ $\frac{N_x}{k_{c,z}Af_c}+\boldsymbol{k_{red}}\frac{M_y}{k_m W_y f_m}\leq 1{,}0$ (11)	$\left(\frac{N_x}{Af_c}\right)^2+\boldsymbol{k_{red}}\frac{M_{y,II}}{W_y f_m}+\frac{M_{z,II}}{W_z f_m}\leq 1{,}0$ $\left(\frac{N_x}{Af_c}\right)^2+\frac{M_{y,II}}{W_y f_m}+\boldsymbol{k_{red}}\frac{M_{z,II}}{W_z f_m}\leq 1{,}0$ (14)	$\frac{-N_x}{A\boldsymbol{f_t}}+\frac{M_{y,II}}{W_y \boldsymbol{f_t}}+\frac{M_{z,II}}{W_z \boldsymbol{f_t}}\leq 1{,}0$ $\frac{-N_x}{A\boldsymbol{f_t}}+\frac{M_{y,II}}{W_y \boldsymbol{f_t}}+\frac{M_{z,II}}{W_z \boldsymbol{f_t}}\leq 1{,}0$ (16)

Figure 11: Overview of the investigated design methods and their consideration of plasticizing under compression and the size effect

Especially in the k_m-method the material specific behaviour is highly simplified. For including the size effect, the coefficient k_{red} is introduced, which increases the bending strength for multiaxial bending. For implementing the elasto-plastic material behaviour under compression in the k_c-method the k_c-factor is adapted accordingly.
For design based on calculations of second order theory the size effect is as well considered with k_{red}. The elasto-plastic material under compression, on the other hand, is taken into account by squaring the compression term.

In order to get an understanding of how the material specific behaviour affects the load-bearing capacity of timber beams at risk of lateral torsional buckling, the analytical results of both effects (eq. (15) and (16) in Figure 11) are compared with the results of purely elastic second order theory calculations (eq. (12) in Figure 11), see diagram in Figure 12. For glued laminated timber (GL 24h), an impact of plastic behaviour under compression can be shown up to a slenderness of $\lambda_{rel,m}$ = 1.00. The size effect, on the other hand, has an influence up to a slenderness of $\lambda_{rel,m}$ = 1.15. For $\lambda_{rel,m}$ = 0.75 there is an increase in the load bearing capacity of about 30 % in comparison to verifications based on purely elastic material behaviour.

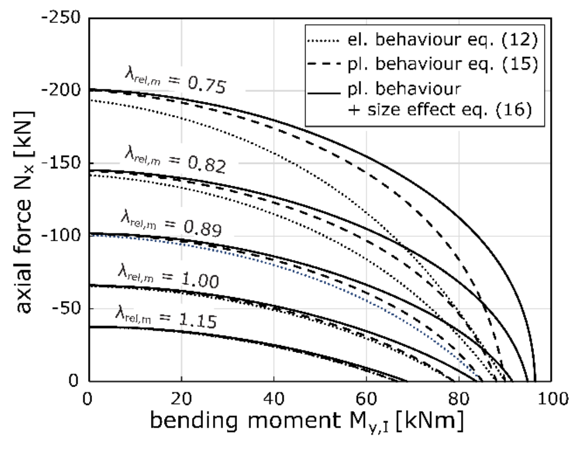


Figure 12: Interaction of bending moment $M_{y,I}$ according to first order theory and axial compression force N_x considering plasticizing (dashed) and additionally the size effect (solid), compared to calculations of linear-elastic second order theory (dotted), depending on the relative slenderness $\lambda_{rel,m}$, based on a verification for stresses of second order theory for lateral torsional buckling

Figure 13 shows the calculation results of the two existing design concepts according to EC5 [5] in comparison to presented verifications based on analytical calculations for three different intermediate slenderness ratios. The curve according to the developed analytical design method shows the highest load-bearing capacity in total. For $\lambda_{rel,m}$ = 0.75, verifications based on second order theory according to EC5 result in a similar graph, however, with a slightly lower load-bearing capacity. Verifications based on calculations of second order theory and on analytical calculations due to the developed design concept result in nearly identical outcome for a slenderness of $\lambda_{rel,m}$ = 1.15.

For combined bending and axial compression, the effective length method generally provides results with a lower load-bearing capacity. However, for pure bending or pure axial compression the maximum bending moment and axial compression force are higher compared to the other two approaches. Also, the graph resulting from the effective length method shows a characteristic kink, where there is a switch in the decisive design equation. Here, the effective length method comes to a higher load-bearing capacity than the other two verification methods.

The higher load-bearing capacity for pure bending and pure axial compression might be explained by the deviation in the assumption of imperfections [17]. The coefficient k_c, which represents the flexural buckling coefficient of the effective length method, implies more favourable imperfections for glued laminated timber (L/1100) than assumed in the other two calculation methods (L/400). Additionally the effective length method for lateral torsional buckling with the coefficient k_m generally neglects the initial torsion of the cross-section.

The kink in the curve of the results of the effective length method occurs due to a switch of the decisive design formula according to DIN 1052 (see Figure 11 – eq. (11a) to (11b)). The cross-section verification of an imperfection-sensitive member is based on a defined Eigenform with the maximum imperfections e_y, e_z and ϑ. According to EC5 [5] and DIN 1052 [16] influences from imperfections in y- and z-direction do not have to be superimposed. Considering e_z (and ϑ), and hence neglecting e_y, bending will cause lateral torsional buckling and an axial compression force will cause flexural buckling around the weak axis. This interaction is represented by equation (11b). Considering e_y, and hence neglecting e_z (and ϑ), buckling around the strong axis is cause by an axial compression force. Here the second order effects due to a bending moment $M_{y,I}$ and e_y are usually negligibly small. A lateral torsional buckling does not occur in this case, since there are no corresponding imperfections (e_z and ϑ).

Equation (11a) thus does not appear to be mechanically justifiable, since combined imperfections e_y, e_z and ϑ are assumed here, but neglecting the proportion of the flexural buckling around the weak axis [1]. In the current revision of EC5 $k_{c,y}$ is replaced by $k_{c,z}$ in equation (11a) [18]. When doing so the effective length method is clearly on the safe side for combined bending moment and axial compression force but mechanically correct.

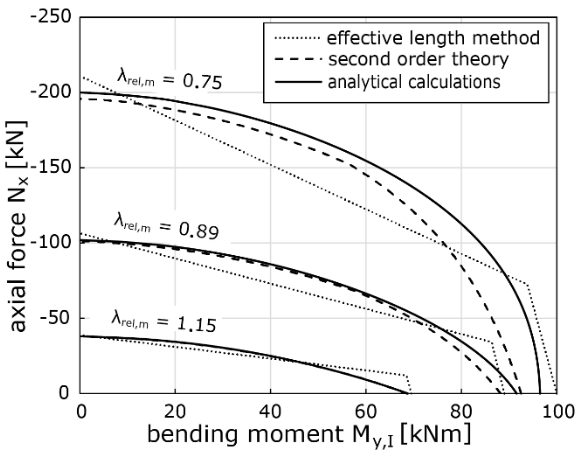


Figure 13: Interaction of bending moment $M_{y,I}$ according to first order theory and axial compression force N_x for verifications based on analytical calculations compared to the effective length method and verifications based on calculations of the second order theory according to EC5 [5], depending on the relative slenderness $\lambda_{rel,m}$

5. Summary

In this paper the results of an investigation of lateral torsional buckling of timber beams subjected to combined bending and axial compression are presented, which was part of a master thesis at the Institute of Structural Design at the University of Stuttgart [4].

For the analytical calculations, approaches for a calculation of internal forces according to second order theory and for an elasto-plastic cross-section failure are combined. Imperfection-sensitive behaviour is described by known solutions of the lateral torsional buckling problem according to second order theory. The cross-sectional failure for combined bending and axial compression is taken into account by a mechanical model developed by *Hörsting* [3], which integrates plasticizing under compression and the size effects on the tensile strength of wood. By combining the two approaches the material specific behaviour of imperfection-sensitive wooden members can be successfully implemented into the analytical calculations.

According to the parametric study, the positive influence of plasticizing under compression and the size effect decreases with increasing slenderness, approaching the ideal-elastic solution according to the calculation of second order theory. For beams of intermediate length, however, there is a notable increase in the load-bearing capacity compared to the design methods of EC5. For $\lambda_{rel,m} = 0.75$ there is an increase in the load bearing capacity of approximately 30 % in comparison to calculations with purely elastic material behaviour. Overall, an impact of the material specific behaviour can be observed up to a slenderness ratio of approximately $\lambda_{rel,m} = 1.00$ to 1.15.

In comparison to the verification based on calculation of second order theory provided by EC5, the own approach based on analytical calculations shows higher load bearing capacities especially for beams of intermediate length. For $\lambda_{rel,m} = 0.75$ there is an increase in the bending moment capacity of approx. 5 %. With increasing slenderness, the interaction curves appear to be nearly identical so that the simplified material behaviour can be seen as a good assumption.
The effective length method generally provides lower load bearing capacities than the results derived by the presented analytical approach, but in some cases the load-bearing capacity of the effective length method is higher. This is due to different assumptions of the equivalent imperfections according to EC5 and shortcomings of the interaction formula [1]. In the 2nd draft of the current revision of the EC5 these deficits are resolved [18]. With these new interaction formulas the results of the equivalent length method for combined bending and compression are far on the safe side compared to verifications based on the presented analytical approach.
When comparing the results for glued laminated timber (GL 24h) to solid timber (C24), higher load bearing capacities could be reached for solid timber, which contradicts practical engineer's understanding. Thus, further investigations are necessary to verify if the implementation of the size effect on the basis of the *Weibull* distribution is satisfying.

Presently there are only very little data regarding experimental tests of imperfection-sensitive timber beams subjected to both bending and axial compression available. For a verification of the analytical model experimental investigations true to scale are necessary. The calculation of internal forces according to second order theory assumes a purely elastic behaviour while the mechanical model according to *Hörsting* [3] describes an elasto-plastic material-behaviour. For coherent results for the verifications based on analytical calculations, the elasto-plastic behaviour should be implemented equally in the second order theory calculations.
Additionally, the assumed failure criterion (eq. 9) only takes into account elastic behaviour on the resisting side as well. Including the elasto-plastic behaviour for calculating plastic moments of resistance ($W_{y,pl}$, $W_{z,pl}$), even bigger load-bearing capacities may be reached.

Regarding the present design methods of EC5, the approach based on the calculation according to second order theory represents a solid simplified method to design slender timber beams subjected to combined bending and axial compression with even a small potential for optimization. For the effective length method further investigations are necessary especially regarding the interaction formula for compression force and bending moment. According to the current revision of EC5 [18] it is suggested to replace the coefficient $k_{c,y}$ by $k_{c,z}$ in equation (11a), so the results may be conservative but mechanically correct and on the safe side.

6. References

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